Journal of International Money and Finance (1991), **10**, 131–148

Optimal dynamic hedging portfolios and the currency composition of external debt

KENNETH F. KRONER

Center for the Study of Futures Markets, Columbia University and Department of Economics, University of Arizona, Tucson, AZ 85721, USA

AND

STIJN CLAESSENS*

World Bank, Washington, DC 20433, USA

We present a model which shows that the currency composition of a country's external debt can serve as a hedging instrument against changes in exchange rates and commodity prices. Because our model permits the second moments to change through time, we get a sequence of optimal dynamic hedging portfolios which can be estimated with a multivariate Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model. To illustrate the usefulness of the technique we apply it to Indonesia and it is found, as expected, that Indonesia's optimal debt portfolio consists of a much larger proportion of US dollars and a much smaller proportion of Japanese yen than they have in their current debt portfolio.

Changes in commodity prices and exchange rates affect firms and countries through several channels. Commodity price changes, for example, affect both exports and imports, and to the extent that exchange rate changes affect commodity prices[1] and trade volumes, they too will affect external trade revenues. Furthermore, if a firm or country has debt obligations in currencies other than their own, then its debt servicing ability will be affected by changes in exchange rates. But the impact of exchange rate and commodity price changes can be minimized through either real diversification (the sourcing, producing and exporting of a mix of products which is optimal given the relationship between exchange rates and goods prices), self-insurance (*e.g.*, reserve management), or financial hedging instruments. The first two options are usually of limited scope to firms given the time and cost it takes to diversify and the opportunity cost of self-insurance, so the hedging tool used most extensively by firms in developed

* We would like to thank Bela Balassa, Amar Bhattacharya, Roger Conover, Rob Engle, Sweder van Wijnbergen, the participants of the Canadian Econometrics Study Group conference and two anonymous referees for comments, though we of course accept full responsibility for any errors ourselves. Ken Kroner would like to thank the Economic Science Lab and the Karl Eller Center, both at the University of Arizona, for financial support. This work was initially undertaken while Ken Kroner was a consultant at the World Bank. The views expresed here do not necessarily represent the views of the World Bank.

0261–5606/91/01/0131–18

countries is financial instruments. Obviously, these financial instruments are also available to countries themselves which are facing large external exposures.

But countries, especially developing countries, may have only limited access to financial markets, due to institutional, credit, and other constraints, and also often lack the expertise necessary to execute short-term hedging strategies with financial instruments. As an alternative, we show in this paper that a country can effectively use the currency composition of external debt as a hedging instrument. In particular, a country can minimize its exposure to exchange rates and commodity price movements by structuring optimally the currency composition of its external debt relative to the costs of servicing the debt. Even though a country may face some constraints in choosing and altering the currency composition of their external debt, this can still lead to risk reduction benefits.

In this paper we derive the debt composition hedging strategy that minimizes the effect of exchange rate fluctuations on a country's terms of trade, and illustrate the potential usefulness of the strategy with an application to Indonesia. In our proposed dynamic hedging strategy, the optimal portfolios depend on the conditional covariance matrix of exchange rates and the terms of trade, which is shown to be changing through time. Therefore, a multivariate extension of the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model of Engle (1982) and Bollerslev (1986) is used to estimate the time-varying debt portfolios.[2] Our results are very promising, with out of sample tests indicating that our proposed strategy would have provided a very effective hedge for the terms of trade against exchange rate exposure for Indonesia.

The structure of the paper is as follows. Section II presents a brief discussion of the analytical model used for the currency management of external debt. Section III describes the data and discusses the econometric technique (GARCH) used in this study, while Section IV applies the model to Indonesia and presents the estimation results and the optimal portfolios. Section IV also contains a discussion and an evaluation of the results. Section V points out some further research directions and concludes.

I. The model

A number of rules have been proposed for choosing the currency denomination of external liabilities.[3] The most favored strategies are to base the currency composition of a country's debt on its pattern of trade, on the currency denomination of its export revenues or on the basket of currencies with respect to which its exchange rate is managed. Most of the alternative suggestions, however, come largely from policy work and are fairly *ad hoc*. The currency composition choice can be better analyzed in the framework of a continuous time portfolio model. We will here develop a portfolio model for a country exposed to exchange rate and commodity price risks.[4]

Consider a world which consists of a small open economy (the home country) and N developed countries. Let each of the N developed countries have an exchange rate e_i, $i=1,\ldots,N$ (measured as home country currency per unit of the foreign currency) which follows the diffusion process

$$\frac{de_i}{e_i} = v_{e_i}(S,t)dt + \sigma_{e_i}(S,t)dZ_{e_i}, \tag{1}$$

where dZ_{e_i} is a Wiener process ($E(dZ)=0$ and $\mathrm{VAR}(dZ)=dt$). We are thus assuming that the exchange rate depreciations are approximately normally distributed for small intervals dt, and that the exchange rates themselves are log normal.[5] The drift $v_{e_i}(S, t)$ and standard deviation $\sigma_{e_i}(S, t)$ are assumed to be functions of time t and the state variables S, where S is a $s \times 1$ vector of state variables which follow Ito processes. The elements of the vector of state variables will be discussed shortly. For notational convenience, we define $y(S, t)$ to be the $N \times 1$ vector of exchange rate depreciations, with ith element de_i/e_i.

The home country can invest its wealth and denominate its liabilities in each of these N currencies. Each country in the 'world' has one nominal riskless (instantaneous) bond. Let B_j^* be the price in the jth currency of country j's riskless bond, and let B be the price in the home currency of the home country's riskless bond. The dynamics for B_j^* are given by

$$\langle 2\rangle \qquad \frac{dB_j^*}{B_j^*} = R_j^* dt, \qquad j = 1, \ldots, N,$$

where R_j^* is the instantaneous nominal rate of return on the jth bond in currency j. Also, let R be the instantaneous nominal return on the safe domestic bond and assume the interest rates R_j^* and R are constant.

Define the excess return of the jth foreign bond for a domestic investor, $[dH(B_j^*)/H(B_j^*)]$, as the return on one unit of domestic currency invested in the foreign bond, financed by borrowing at the interest rate R in the domestic country. That is,

$$\langle 3\rangle \qquad \frac{dH(B_j^*)}{H(B_j^*)} = \frac{dB_j^*}{B_j^*} + \frac{de_j}{e_j} - Rdt$$

$$= (R_j^* + v_{e_j}(S, t) - R)dt + \sigma_{e_j}(S, t)dZ_{e_j} \qquad j = 1, \ldots, N.$$

Clearly, the foreign bonds are risk-free in their own country but exchange rate risks make them risky for investors from our 'home country.' For notational convenience, we will use $\eta(S, t, R, R^*)$ to represent the $N \times 1$ vector of excess returns. Also, notice that because R_j^* and R are constants, the excess returns are perfectly correlated with the changes in the corresponding exchange rate, *i.e.*, $\mathrm{CORR}(y_i, \eta_i) = 1$.

Next, suppose there is one commodity consumed in the home country, whose domestic currency price follows the differential equation:[6]

$$\langle 4\rangle \qquad \frac{dP}{P} = v_p(S, t)dt + \sigma_p(S, t)dZ_p.$$

Again, $v_p(S, t)$ and $\sigma_p(S, t)$ are allowed to be functions of both a vector of state variables and time. The first element in the $S \times 1$ vector of state variables is the change in the (logarithm of the) price and the next N elements are the changes in the (logarithms of the) exchange rates, so $S = [(dP/P), (de_1/e_1), \ldots, (de_N/e_N)]$. The price P represents the price of servicing external debt relative to domestic consumption and can therefore best be interpreted as the terms of trade of the country—*i.e.*, the export price divided by the import price.

Finally, we assume that the country's welfare problem can be reduced to finding the currency composition of its external debt that minimizes the variance of its external debt service (*i.e.*, the excess return to the foreign bonds) relative to its opportunity cost of consumption (*i.e.*, the terms of trade).[7] In other words, the country seeks to minimize the variability of its debt service relative to a measure of its foregone consumption. Defining b to be the $N \times 1$ vector of optimal holdings of foreign bonds, then, the country's objective function is

$$\langle 5\rangle \qquad \min_b \operatorname{var}\left(b'\eta(S,t,R,R^*) - \frac{dP}{P}\right).$$

To simplify this equation, let $\Omega_{\eta\eta}(S,t)$ be the $N \times N$ conditional covariance matrix of excess returns to the foreign bonds, and let $\Omega_{\eta p}(S,t)$ be the $N \times 1$ vector of conditional covariances between excess returns and percentage changes in the price variable. Notice from equation ⟨3⟩ that, because excess returns are perfectly correlated with exchange rate depreciations, $\Omega_{\eta\eta}(S,t)$ is the same as the conditional covariance matrix of exchange rate depreciations, *i.e.*, $\Omega_{\eta\eta}(S,t) = \Omega_{yy}(S,t)$, and $\Omega_{\eta p}(S,t)$ is the same as the matrix of conditional covariances between the exchange rate depreciations and percentage changes in the price variable, *i.e.*, $\Omega_{\eta p}(S,t) = \Omega_{yp}(S,t)$. Then the problem becomes

$$\langle 6\rangle \qquad \min_b \left(b'\Omega_{yy}(S,t)b - 2b'\Omega_{yp}(S,t) + \sigma_p(S,t)\right)$$

and the optimal holdings of foreign bonds $b^*(S,t)$ is

$$\langle 7\rangle \qquad b^*(S,t) = \Omega_{yy}(S,t)^{-1}\Omega_{yp}(S,t).$$

The conclusion from this model, then, is that the optimal risk-minimizing currency composition is a function of the conditional covariances of the exchange rate depreciations and the conditional covariance of each of the exchange rate depreciations with the price variable. These are all permitted to change with time, suggesting that a correct implementation of the model requires an estimation method which permits time-varying variances and covariances. The hedging portfolio provides the best hedge against changes in the exchange rates by finding the portfolio that has the maximum correlation with the percentage changes in the state variable. The resulting borrowing shares would apply to the country's net foreign liabilities, *i.e.*, gross debt minus foreign exchange reserves. Positive elements of the vector $b^*(S,t)$ indicate optimal borrowing shares, while negative elements indicate asset shares. We now turn our attention to a discussion of the data and the method used to estimate ⟨7⟩.

II. Data and estimation technique

Weekly exchange rate data from April 30, 1980 to March 31, 1988 were used (414 observations) for the following eight exchange rates: the Japanese yen (JY), the Deutschemark (DM), the Swiss franc (SWF), the Austrian schilling (AUS), the pound sterling (PS), the French franc (FF), the US dollar (US), and the Indonesian rupiah (INDO)[8]. Weekly exchange rates of the developed countries' currencies were calculated in terms of number of Indonesian rupiahs per unit of foreign currency (*e.g.*, rupiahs per pound sterling), giving a set of seven exchange rates ($N = 7$).

A few interesting facts are obvious from the plots of these exchange rates. For a typical plot, see Figure 1, which contains the INDO/US exchange rate, or

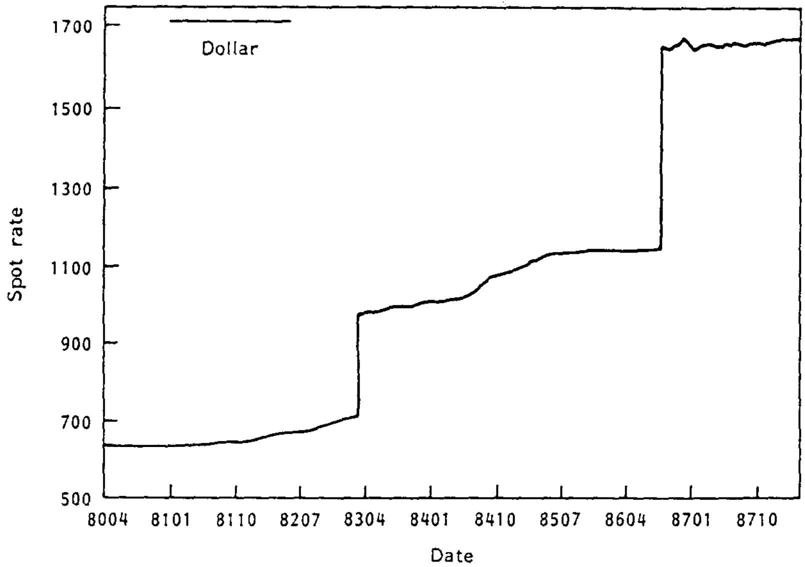

FIGURE 1. US dollar rate.

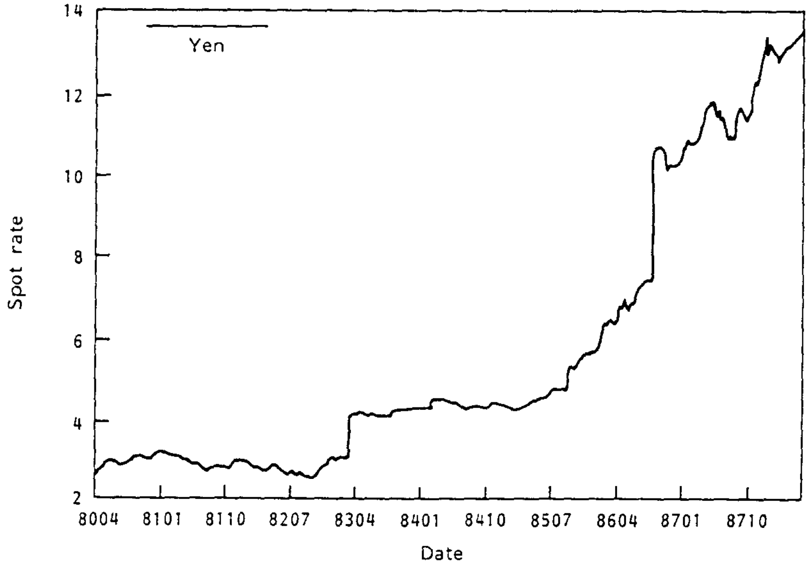

FIGURE 2. Japanese yen rate.

Figure 2, which plots the INDO/JY exchange rate. The most striking feature of these plots is the dramatic depreciation of the rupiah, falling, for example, from 629 INDO/US at the beginning of the sample to 1660 INDO/US at the end of the sample. Another obvious feature is that the Indonesian government devalued the rupiah two times by relatively large amounts—from 702 INDO/US to 970 INDO/US in March 1983 and from 1134 INDO/US to 1643 INDO/US in September, 1986. These two devaluations of the rupiah could cause problems when estimating the optimal hedge ratios because the large appreciations of all currencies at the same time would artificially increase covariance estimates. A dummy variable is therefore used throughout the ensuing analysis to capture each of these two depreciations (one dummy variable for each depreciation). One final observation is the smoothness of the US rate compared to the JY rate, making it quite clear that the Indonesian rupiah was managed with respect to the dollar.

Several preliminary diagnostic checks on the distributional properties of the exchange rates are conducted. First, nonstationarity (*i.e.*, the presence of a unit root) in each of the exchange rates is verified using the Phillips–Perron (1988) tests. The Phillips–Perron tests are tests for unit roots which are robust to the kinds of non-linear dependencies that are common to foreign exchange markets. See Table 1 for the results and the 95 per cent critical values. Significance at 95 per cent is indicated with an asterisk. One observation that is clear from this table is that the null hypothesis of a unit root in each of the exchange rates cannot be rejected, suggesting that the exchange rates are all random walks (possibly with trend). This is somewhat surprising, given the interventions in the market for rupiahs during the sample period by the Indonesian authorities, but it does suggest that the effects of the interventions were unpredictable.

Further analysis reveals that the differenced data are highly leptokurtic, which is a property common to foreign exchange data (see Bollerslev *et al.*, 1991); see Table 2. In fact, as expected the Bera–Jarque test for normality, which is distributed χ^2_2 under the null (see Bera and Jarque, 1982), is highly significant for all currencies. The unconditional distribution from a conditionally normal ARCH model is known to be leptokurtic (see Engle, 1982), suggesting that an ARCH model might be able to capture some of this high kurtosis. The Box–Pierce $Q(8)$ ($\sim\chi^2_8$) and Durbin–Watson tests for serial correlation are generally insignificant,[9] while the test for fourth order ARCH ($\sim\chi^2_4$) is generally significant. The conclusion from these tests is that an ARCH model on first differences is a good place to start when modelling each of the exchange rates used here.

The analysis in this paper focuses on the relationship between the countries' exchange rates and terms of trade, which are calculated as the unit value of exports divided by the unit value of imports. It should be noted that the percentage changes from month to month in the terms of trade are quite high compared to the monthly exchange rate changes, which are much less volatile and usually fluctuate between ± 10 per cent. Also, the correlations between the exchange rate changes and the terms of trade changes are relatively low, never exceeding 0.16, suggesting that the optimal portfolio we derive might be a less than perfect hedge against exchange rate fluctuations.

We are interested in estimating equation $\langle 7 \rangle$, where Ω_{yp} is the vector of conditional covariances between the changes in the terms of trade and the changes in the exchange rates and Ω_{yy} is the covariance matrix of exchange rate depreciation rates. Notice that if covariances were constant through time then $\Omega_{yy}^{-1}\Omega_{yp}$ is just a simple OLS regression of the changes in the terms of trade on changes in the exchange rates, and one could calculate the optimal portfolio shares by running a simple OLS regression of the terms of trade changes on the exchange rate changes and interpreting the OLS parameter estimates as relative portfolio weights. However, it is shown in Table 2 above that ARCH is significant—*i.e.*, the conditional variances are not constant through time—so an estimation procedure which allows the covariance matrix to change with time should be used.

TABLE 1. Phillips–Perron tests for unit roots in log-exchange rate data.

$$Y_t = \hat{\alpha} Y_{t-1} + \hat{u}_t$$
$$Y_t = \mu^* + \alpha^* Y_{t-1} + u_t^*$$
$$Y_t = \tilde{\mu} + \tilde{\beta}(t - T/2) + \tilde{\alpha} Y_{t-1} + \tilde{u}_t$$

Statistic	Null hypothesis	ln(JY)	ln(DM)	ln(SWF)	ln(AUS)	ln(PS)	ln(FF)	ln(US)	95% cv's
$Z(t_{\hat{\alpha}})$	$\hat{\alpha} = 1$	3.15	1.41	1.56	1.49	0.73	0.41	4.54	−1.95
$Z(t_{\alpha^*})$	$\alpha^* = 1$	1.36	0.93	0.80	0.98	0.06	−0.23	−0.38	−2.86
$Z(\phi_1)$	$\alpha^* = 1;\ \mu^* = 0$	5.07*	1.36	1.45	1.48	0.27	0.12	10.45*	4.59
$Z(t_{\tilde{\alpha}})$	$\tilde{\alpha} = 1$	−0.69	−0.99	−1.02	−0.97	−0.28	−0.80	−1.01	−3.41
$Z(\phi_2)$	$\tilde{\alpha} = 1;\ \tilde{\beta} = 0$	4.25	3.61	2.80	3.57	2.34	3.42	7.36*	6.25
$Z(\phi_3)$	$\tilde{\alpha} = 1;\ \tilde{\beta} = \tilde{\mu} = 0$	2.19	4.51	3.07	4.38	3.25	5.04	0.52	4.68

TABLE 2. Preliminary data analysis on log-differenced exchange rate data.

Statistic	JY	DM	SWF	AUS	PS	FF	US
Skewness	0.80	0.37	0.48	0.39	0.35	0.01	0.29
Kurtosis	4.85	3.57	3.76	3.71	5.17	4.79	4.80
Bera–Jarque	103.8*	14.78*	25.80*	18.95*	90.08*	55.35*	61.60*
Box–Pierce $Q(8)$	12.27	12.29	9.29	10.79	15.37	13.22	90.88*
Durbin–Watson	1.75	1.87	1.91	1.87	1.93	1.85	1.34*
ARCH(4)	13.00*	21.53*	13.66*	20.45*	22.36*	6.91	57.71*

A multivariate generalization of the Autoregressive Conditional Heteroskedasticity (ARCH) model of Engle (1982) should be ideally suited to this purpose. The univariate ARCH model allows the current conditional variance of a time series to depend on lagged squared residuals in an autoregressive manner. This means that during periods in which there are large unexpected shocks to the variable its estimated variance will increase, and during periods of relative stability its estimated variance will decrease. Bollerslev (1986) generalizes the ARCH model (to GARCH) by allowing the current conditional variance to depend on lagged conditional variances as well as lagged squared residuals. In effect, then, the GARCH model is similar to an ARMA model in squared residuals. The generalization of univariate GARCH models to multivariate GARCH models requires allowing the whole covariance matrix to change with time, instead of allowing just the variance to change with time. This is usually done by allowing all the elements of the covariance matrix to be linear functions of lagged squares and cross products of the residuals and lagged variances and covariances (see Baba *et al.*, 1989; or Bollerslev *et al.*, 1988. So this generalization is similar to the generalization of a univariate ARMA process to a vector ARMA process.

Alternatively, Diebold and Nerlove (1989) propose another specification of the multivariate GARCH process which is similar to the traditional factor analysis model except that the underlying factors follow GARCH processes, while Bollerslev (1990) develops yet a third multivariate GARCH specification which imposes the restriction that the correlation matrix is constant through time. The Bollerslev (1990) model, while more restrictive than the kind discussed above, is simpler and much easier to estimate. By imposing the restriction that the correlation matrix is constant through time while allowing the variances to follow univariate GARCH processes, this model allows the whole covariance matrix to change through time. The constant correlations model has been applied successfully to foreign exchange rate data by Baillie and Bollerslev (1990), Bollerslev (1990), and Giovannini and Jorion (1989), among others, and to interest rate data by Cecchetti *et al.* (1988). Giovannini and Jorion (1989), in fact, show that the estimated variances from the constant correlations model are almost perfectly correlated with those from the vector ARMA-type models, and Baillie and Bollerslev (1990) and Bollerslev (1990) show that, while the constant correlations assumption might seem to be highly restrictive, it is typically not rejected by the data. Because of its computational simplicity, then, we use the constant correlations model in this paper.

III. Estimation results and optimum portfolios

Focussing first on the estimation of $\Omega_{yy}(S, t)$, let y_t be the 7×1 vector of exchange rate depreciations, let Ω_t be the 7×7 covariance matrix of exchange rate depreciations, and let D_{1t} be a dummy variable for the first major depreciation in the rupiah and D_{2t} be a dummy variable for the second major depreciation. Then the multivariate ARCH model is

$$\langle 8 \rangle \qquad \begin{aligned} y_t &= \alpha + \delta_1 D_{1t} + \delta_2 D_{2t} + \varepsilon_t \\ \varepsilon_t | F_{t-1} &\sim N(0, \Omega_t) \\ \Omega_t &= V_t C V_t . \end{aligned}$$

Here, F_t (the information set) is the σ-field generated by past values of ε_t. Also, α, δ_1, δ_2, and ε_t are all 7×1 vectors, C is a 7×7 time invariant correlation matrix, and V_t is a 7×7 diagonal matrix in which the ith diagonal element, $\sigma_{i,t}$, is the conditional standard deviation of the ith exchange rate depreciation. The variances are assumed to follow univariate GARCH(1,1) processes: *i.e.*, $\sigma^2_{i,t} = \omega_i + a_i \varepsilon^2_{i,t-1} + g_i \sigma^2_{i,t-1}$, $i = JY, DM, \ldots, US$. This gives a covariance matrix, Ω_t, with constant correlations but time-changing variances and covariances. To ensure that the effect of each of the two large depreciations is the same on all seven exchange rates (in terms of percentages), restrictions are imposed that $\delta_1 = r_1 \iota$ and $\delta_2 = r_2 \iota$, where r_1 and r_2 are scalars and ι is a (7×1) vector of ones.

Letting θ be the parameters of the model and T be the number of observations, the likelihood function is

$$L(\theta) = -\frac{Tn}{2} \log(2\pi) - \frac{1}{2} \sum_{i=1}^{T} (\log|\Omega_t| + \varepsilon_t' \Omega_t^{-1} \varepsilon_t),$$

which can be written as

$$L(\theta) = -\frac{Tn}{2} \log(2\pi) - \tfrac{1}{2} T \log|C| - \sum_{i=1}^{T} \log|V_t| - \frac{1}{2} \sum_{i=1}^{T} s_t' C^{-1} s_t ,$$

where s_t are the standardized residuals. The parameter values are obtained by maximizing the likelihood function using numeric techniques[10], and these parameter values can be used to estimate (and forecast) time changing conditional covariance matrices. These conditional covariance matrices are then used for $\Omega_{yy}(S, t)$ in equation $\langle 7 \rangle$ to find the optimal time-varying debt portfolios.

The results from the estimation of model $\langle 8 \rangle$ are presented in Table 3 (t-stats in parentheses). The first observation on these results is that the ARCH parameters a and g are almost always highly significant, which suggests that the variances and covariances are changing through time and the ARCH estimation procedure should give us better covariance estimates at any point in time than OLS. Another observation is that the constants in the mean equations are usually significantly positive, capturing the upward trend in the exchange rates. Notice, for example, that α_{DM} is 0.199, meaning that the average weekly depreciation of the rupiah relative to the Deutschemark during the sample was about 0.199 per cent. The depreciation dummies imply that the first depreciation was about 32 per cent and the second was about 37 per cent.

One further observation from Table 3 is that some of the currencies (in particular JY, SWF, and US) appear to be integrated or nearly integrated in

TABLE 3. ARCH estimation results—data through March 1988.

Parameter	JY	DM	SWF	AUS	PS	FF	US
α	0.327	0.199	0.237	0.204	0.091	0.104	0.029
t-stats	(4.6)	(2.6)	(2.9)	(2.7)	(1.2)	(1.2)	(5.3)
w	0.130	0.470	0.199	0.483	0.112	0.636	0.000
t-stats	(3.5)	(2.1)	(1.5)	(2.1)	(2.1)	(1.9)	(1.5)
a	0.072	0.058	0.035	0.056	0.047	0.098	0.292
t-stats	(5.7)	(2.4)	(2.6)	(2.2)	(2.9)	(4.4)	(5.8)
g	0.863	0.734	0.892	0.729	0.902	0.634	0.739
t-stats	(30.7)	(6.4)	(16.0)	(6.2)	(25.3)	(3.4)	(23.4)

Notes
Dummy variables: Depreciation No. 1 32.26
(163)
Depreciation No. 2 37.21
(531)

variance (see Engle and Bollerslev, 1986), opening up the possibility of copersistence in variance (see Bollerslev and Engle, 1989). If the variances are copersistent, then a model similar to cointegration in variance would be the appropriate way of modelling the covariance matrix. To examine this possibility we conduct Wald tests for integration (or persistence) in variance, *i.e.*, $a_i+g_i=1$, against the alternative $a_i+g_i<1$. The test results, which are shown in Hong (1988) to be asymptotically normal, are reported in the first row of Table 4. With a 95 per cent critical value of -1.645, we find an explosive root in US, weak evidence of persistence in SWF and no evidence for persistence in any of the other currencies. The conclusion, then, is that copersistence is highly unlikely. It should be recognized that this result is probably a consequence of our weekly observation interval; persistence is usually evident in higher frequency data. See, for example, Baillie and Bollerslev (1989) or Hsieh (1989).

Other diagnostic tests are also reported in Table 4. In particular, the results of Lagrange Multiplier (LM) tests for higher order GARCH specifications are given in rows 2 and 3. Row 2 contains the results of tests for GARCH(1,2) models (*i.e.*, for the inclusion of $\varepsilon_{i,t-2}^2$ in the ith GARCH equation), and row 3 presents the results of tests for GARCH(2,1) models (*i.e.*, for the inclusion of $\sigma_{i,t-2}^2$ in the ith GARCH equation). At first glance, it appears that higher order models are appropriate for some of the currencies. However, the standardized residuals from the model exhibit excess kurtosis (row 4) which causes the LM statistics to be overstated (see Bollerslev and Wooldridge, 1990). In fact, Bollerslev and Wooldridge (1990) show that when mild conditional leptokurtosis (conditional kurtosis $=6$) exists, the empirical size of the LM test for GARCH(1,2) used here is 16 per cent. This suggests that the true critical values are much larger than the 3.84 used in Table 4, implying that the statistics which appear to be marginally significant in rows 2 and 3 probably are not significant, but instead are caused by the conditional leptokurtosis. Fortunately, though, the parameter estimates themselves are still consistent and are unaffected by the presence of excess conditional kurtosis (see Bollerslev and Wooldridge, 1990).

The correlation matrix of weekly exchange rate depreciations (*i.e.*, estimates

TABLE 4. Diagnostic tests.

Test	JY	DM	SWF	AUS	PS	FF	US
IGARCH	−3.11*	−2.15*	−1.60	−2.21*	−2.13*	−1.82*	+1.03
GARCH(1,2)	5.69*	0.34	0.26	7.70*	0.49	4.06*	3.16
GARCH(2,1)	7.58*	1.64	5.47*	4.21*	3.07	3.90*	3.28
Stand. kurt.	4.77	3.30	3.42	3.43	3.90	4.63	3.43

TABLE 5. Pearson correlation matrix for Indonesian-based exchange rate depreciations.

JY	DM	SWF	AUS	PS	FF	US
1.00	0.648	0.701	0.649	0.460	0.596	−0.477
—	(22)	(27)	(21)	(11)	(15)	(−10)
	1.00	0.919	0.990	0.649	0.893	−0.429
	—	(112)	(949)	(23)	(91)	(−9)
		1.00	0.917	0.627	0.835	−0.444
			(114)	(21)	(51)	(−10)
			1.00	0.660	0.901	−0.422
			—	(24)	(106)	(−9)
				1.00	0.660	−0.335
				—	(21)	(−6)
					1.00	−0.375
					—	(−7)
						1.00
						—

of the matrix C) is given in Table 5.[11] Notice that the correlations of the US dollar with all other exchange rates are much lower than the correlations between any other exchange rates, and in fact they are negative. Notice also that the European currencies (DM, SWF, AUS, and FF) form a block with high correlations, and in fact, the DM and AUS are almost perfectly correlated. This suggests that the AUS adds no information (and no hedging potential) beyond that already given by the DM. For this reason, the AUS is dropped from the ensuing analysis.[12]

From these results, a series of conditional variances can be constructed which allows us to identify periods of stability and instability in each of the exchange rates. The conditional variances of the US dollar rate and the pound sterling rate (*i.e.*, $\sigma^2_{US,t}$ and $\sigma^2_{PS,t}$) are plotted in Figure 3.[13] Some interesting insights can be gained from a study of this plot. First, the variances of the US dollar rate are much smaller than the variances of the other series. This is expected because the rupiah is being managed with respect to the dollar. Notice also that uncertainty in most exchange rates was at a peak in 1985—a time when uncertainty in the dollar was relatively low. One possible explanation for this is that the market knew that the dollar was going to fall, but there was a lot of uncertainty in how the fall would affect other currencies.

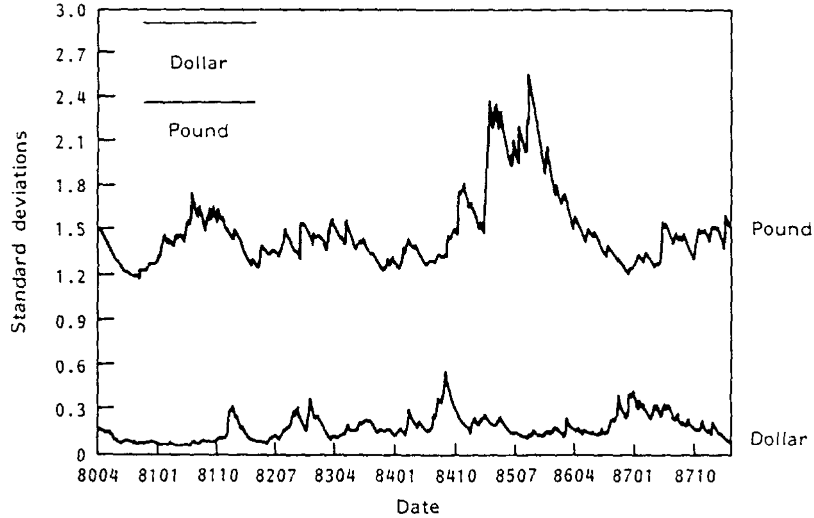

FIGURE 3. Conditional standard deviations.

TABLE 6. Lagrange multiplier statistics for ARCH in the covariances between the changes in the terms of trade and currency depreciations.

JY	DM	SWF	PS	FF	US
0.45	0.05	0.04	0.04	0.01	10.41*

We now turn our attention to estimating $\Omega_{yp}(S, t)$ in equation $\langle 7 \rangle$. Terms of trade data are only available monthly, which means that monthly covariances must be used. Tests are undertaken to see whether or not these covariances also follow an ARCH process, using a test statistic derived in Appendix 2. Under the null hypothesis that there is no ARCH in the monthly covariances between exchange rate depreciations and percentage changes in the terms of trade, the test statistic is distributed χ_1^2, with a 5 per cent critical value of 3.84. Table 6 presents the computed test statistics. The only significant test statistic is for the covariance between changes in the US dollar and the changes in the terms of trade. Because all other test statistics are highly insignificant, we will assume that the vector $\Omega_{yp}(S, t)$ is constant through time and subsequently use the unconditional covariances between exchange rates and terms of trade for $\Omega_{yp}(S, t)$ in equation $\langle 7 \rangle$.

Finally, to calculate a nation's optimal debt portfolio over the next (say) three months, we use the GARCH model $\langle 8 \rangle$ to forecast the variance–covariance matrix of exchange rate depreciations for the next three months, and then multiply the inverse of that by the three-month forecast of the covariance between exchange rate depreciations and changes in the terms of trade (see equation $\langle 7 \rangle$). The three-month forecast of $\Omega_{yy}(S, t)$ is obtained by summing the 13 one-week forecasts derived from model $\langle 8 \rangle$, and the three-month forecasts of $\Omega_{yp}(S, t)$ are found by

TABLE 7. Forecasted covariance matrix—1988.1.

JY	DM	SWF	PS	FF	US
28.8	19.0	22.6	12.6	17.2	−1.3
	30.1	30.5	18.4	26.7	−1.2
		36.7	19.6	27.5	−1.4
			26.9	18.5	−0.9
				29.6	−1.0
					0.26

TABLE 8. Unconditional covariance matrix—1980.2 to 1987.4

JY	DM	SWF	PS	FF	US
36.7	20.8	26.1	17.0	21.3	−1.1
	33.0	32.8	19.6	31.5	−0.97
		39.3	22.0	31.2	−1.1
			31.0	20.3	−0.9
				33.9	−0.9
					0.69

multiplying the unconditional monthly covariances by three. Tables 7 and 8 present the forecasted conditional covariance matrix for the first quarter of 1988 and the sample covariance matrix over the whole sample period (Apr/80 to Mar/88), respectively. The two covariance matrices are quite different, suggesting that the exchange rates in the first quarter of 1988 were expected to be relatively more stable than over the previous eight years combined.

With this information, the optimal forecasted portfolio for the first quarter of 1988, scaled to sum to one, can be calculated, giving

JY	DM	SWF	PS	FF	US
0.031	0.191	−0.005	0.014	−0.139	0.907.

The most striking feature of this portfolio is the heavy weight in the US dollar. This is not surprising because Indonesia's exports are largely made up of petroleum and primary commodities, whose prices are closely related to the US dollar, and because Indonesia manages its exchange rate with respect to the US dollar. Therefore, borrowing a large fraction in US dollars should provide a hedge for changes in terms of trade against currency fluctuations.

The optimal forecasted portfolio for the first quarter of 1986 (*i.e.*, based only on data up to the end of 1985, or the first 297 observations) can be calculated in a similar way, giving

JY	DM	SWF	PS	FF	US
−0.005	0.307	−0.055	0.007	−0.154	0.900.

This is not remarkably different from the optimal portfolio for the first quarter of 1988, but it differs substantially from Indonesia's actual debt portfolio at the end of 1985, which was

JY	DM	SWF	PS	FF	US
0.401	0.106	0.062	0.025	0.038	0.369.

Given the substantial difference between the optimal portfolio and the actual portfolio, it would seem that the optimal portfolio should result in a dramatic improvement in Indonesia's ability to hedge themselves against currency fluctuations.

In order to evaluate how effective this portfolio strategy is in terms of dynamically hedging the terms of trade against exchange rate fluctuations, we assume that the portfolio was adjusted optimally every quarter since the end of 1985, using only the information available at the beginning of the relevant quarter. This gives the sequence of optimal forecasted portfolios depicted in Table 9, where the portfolios are again scaled to sum to one. The relative shares of the currencies change from quarter to quarter due to the changing conditional covariances, but the effective currency distribution of the portfolios does not change much through time once one accounts for the high correlation between the European currencies. The sums of the shares of the European currencies (DM, SWF, and FF) for each of the respective quarters are 9.8 per cent, 11 per cent, 5.2 per cent, 15.1 per cent, 19.5 per cent, 12.3 per cent, 20.7 per cent, 8.1 per cent, and 4.7 per cent, suggesting that the combined European share is more stable than the individual shares. In addition to the changes in shares, the unscaled portfolios also change. The sum of the unscaled portfolio weights ranges between about 5 and about 40, which suggests different optimal absolute levels of borrowing.

From these portfolios the monthly sequence of $b * \Delta \ln E - \Delta \ln P$ can be calculated for 1986 and 1987, where E are the residuals from the exchange rate equations, and b are the portfolio weights. Assuming borrowing at the absolute levels implied by the optimal portfolio strategy, the variance of this sequence can be compared with the variance of the sequence which results when Indonesia uses its 1985 portfolio composition throughout the two years. Comparison of the variances of the two portfolios provides an indication of how well the optimal strategy hedges against exchange rate exposure. Performing this exercise shows that the variance drops significantly (by 56 per cent) using the optimal strategy.

TABLE 9. Optimal portfolios.

Period	JY	DM	SWF	PS	FF	US
1986.1	−0.005	0.307	−0.055	0.007	−0.154	0.900
1986.2	−0.022	0.320	−0.028	0.028	−0.182	0.884
1986.3	−0.001	0.164	−0.012	0.021	−0.100	0.928
1986.4	−0.027	0.384	0.019	0.027	−0.252	0.849
1987.1	−0.009	0.801	0.026	0.150	−0.632	0.665
1987.2	0.006	0.462	0.015	0.075	−0.354	0.797
1987.3	−0.033	0.703	−0.017	0.050	−0.479	0.777
1987.4	0.044	0.323	0.001	0.029	−0.243	0.847
1988.1	0.031	0.191	−0.005	0.014	−0.139	0.907

Evidently, the movement in Indonesia's borrowing portfolio away from Japanese yen to US dollars results in the increased stability in the relative cost of servicing its external debt.

Similar analyses can be conducted to find portfolios that hedge against changes in export prices, export values, import prices, and import values. The resulting portfolios are similar to those above and equally large reductions in variance are achieved. The import hedging portfolios are approximately the negative of the export hedging portfolios while the export hedging portfolios are approximately the same as the terms of trade portfolios.

IV. Conclusions

This paper outlines a theoretical model providing a way to calculate the optimal debt portfolio for a nation which wants to hedge its terms of trade against exchange rate fluctuations, and applies the model to Indonesia. The resulting portfolios are shown to be very effective hedges, so even though Indonesia might only have limited access to organized currency futures and other hedging markets, they could still manage their external exposure effectively by structuring their external debt optimally. Furthermore, the proposed strategy seems particularly feasible because the optimal portfolios are fairly stable over time, implying that in practice this strategy would require mostly fine-tuning their debt portfolio.

Appendix 1: data

The source for INDO was the International Monetary Fund database TIBMER, and the source for the other rates was the IMF database FTFROR. The Indonesian rate is a 'representative' rate—*i.e.*, it comes from markets within Indonesia. The other rates are all London Noon Spot Quotations. Wednesdays were used whenever possible, but if a holiday fell on Wednesday then Thursday's quotation was used.

Value and export volume data come from the International Financial Statistics. The source for the import volume data is the *Indikator Ekonomi*, a monthly publication of the Indonesian Bureau of Statistics. The unit values of exports (imports) are calculated by dividing the export (import) values by the export (import) volumes.

Appendix 2

When testing for restrictions in the ARCH models, the simplest test is the Lagrange Multiplier (LM) test. This is a test which examines whether the slope of the likelihood function, evaluated at the parameters under the null hypothesis, is zero. So the LM test requires only the derivatives of the likelihood function with respect to all the parameters, evaluated under the null. If the null hypothesis is H_0: $\theta = \theta_0$ then the LM test statistic, which is distributed χ^2_q where q is the number of restrictions imposed by the null, is

$$\xi_{LM} = \left(\frac{\partial L}{\partial \theta}\right)' \hat{\Psi}^{-1} \left(\frac{\partial L}{\partial \theta}\right)\Bigg|_{\theta=\theta_0},$$

where Ψ is the information matrix and L is the likelihood function.

Kroner (1987) shows that for ARCH models this test statistic always reduces to

$$\xi_{LM} = \frac{1}{2}\left(\sum_{i=1}^{T} v_t'\Gamma_t^{-1}R_t\right)'\left(\sum_{i=1}^{T} R_t'\Gamma_t^{-1}R_t\right)\left(\sum_{i=1}^{T} R_t'\Gamma_t^{-1}v_t\right)\Bigg|_{\theta=\theta_0}$$

where

$$v_t = \text{vec}(\varepsilon_t \varepsilon_t' - \Omega_t)$$

$$\Gamma_t = \Omega_t \otimes \Omega_t$$

and

$$R_t' = \frac{\partial \text{ vec } \Omega_t}{\partial \theta}.$$

In this paper, we are interested in testing whether or not the covariances between exchange rate depreciations and terms of trade changes are varying with time. This is done by setting up a bivariate seemingly unrelated regressions model, with the two variables being the appropriate exchange rate depreciations and the changes in the terms of trade. We then test for ARCH in the covariances from this regression. So if the model is

$$\Omega_t = \Omega + \begin{bmatrix} 0 & p\varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ p\varepsilon_{1,t-1}\varepsilon^2{}_{,t-1} & 0 \end{bmatrix},$$

then the null hypothesis for no ARCH in the covariances is H_0: $p=0$. The parameters θ are $\theta = (\text{vec }\Omega, p)$, and R_t is given by

$$R_t = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & p\varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ 0 & 0 & 1 & 0 & p\varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}_{4\times 5}$$

This test statistic is distributed as a χ_1^2 under the null of no ARCH in the covariances.

Notes

1. As, for instance, in the (historically inverse) relationship between commodity prices and the value of the dollar. See Dornbusch (1987).
2. For applications of multivariate GARCH models to more traditional portfolio hedging problems in finance, see Cecchetti *et al.* (1988), Baillie and Myers (1991), and Kroner and Sultan (1991).
3. See, for example, Lessard and Williamson (1985).
4. For other models of international portfolio choice see Adler and Dumas (1983), Claessens (1988), Stulz (1981), and Svensson (1987). For a more detailed discussion of a model similar to the one presented here, see Claessens (1988).
5. See Merton (1971) and Fischer (1975) for more detailed descriptions of the properties of Weiner processes and stochastic differential equations.
6. The use of one price variable instead of multiple variables can be justified if the utility function to be maximized exhibits constant consumption shares. Note that we assume neither that the law of one price nor purchasing power parity holds *vis-à-vis* all currencies, *i.e.*, P is not necessarily equal to $P_j^* e_j$ for all j, where P_j^* is the price of the good in terms of the foreign currency j. Neither do we assume that changes in the terms of trade are perfectly correlated with the (weighted average of the) changes in the exchange rates.
7. In general the optimal portfolio will consist of a hedging and speculative component, where the latter will depend on expected excess costs of liabilities in different currencies (see Claessens, 1988). The expected excess costs (*ex ante* deviations from uncovered interest rate parity) will be determined by risk premiums and will be largely determined in the developed capital markets. To the extent that the country is more risk averse than the world, risk premiums will be small relative to its risk preferences, and the hedging component will be the most important. We will concentrate therefore on the hedging portfolio.

8. The rupiah was fixed with respect to the US dollar in the 1970s. This would affect the estimate of the variances and covariances of the exchange rates, so data were used starting with implementation of the managed float exchange rate system on April 30, 1980. See Appendix 1 for data sources.
9. It should be recognized that the Box–Pierce and Durbin–Watson statistics tend to over-reject when ARCH is present, suggesting that part of the appearance of serial correlation in the US dollar might actually be due to serial dependence in the second moments—*i.e.*, due to ARCH. See Diebold (1988).
10. We would like to thank Tim Bollerslev for graciously supplying his program.
11. It is interesting to note that without the dummy variables, all the correlations increased dramatically. For example, the correlations involving the US dollar increased from about −0.40 to about +0.80.
12. To support the conjecture that AUS adds little hedging potential, it should be noted that dropping AUS only causes a redistribution of the holdings of the European currencies in our optimal portfolio, leaving the portfolio weights on the non-European currencies (JY, PS, and US) almost unaffected.
13. These are just a representative sample of all the series which could be presented; the conclusions derived from these are similar to those which would be derived from the other series.

References

ADLER, M., AND B. DUMAS, 'International Portfolio Choice and Corporation Finance: A Synthesis,' *Journal of Finance*, June 1983, **38:** 925–984.

BABA, Y., R.F. ENGLE, D.F. KRAFT, AND K.F. KRONER, 'Multivariate Generalized Autoregressive Conditional Heteroskedasticity,' unpublished manuscript, UC San Diego, 1989.

BAILLIE, R.T., AND T. BOLLERSLEV, 'The Message in Daily Exchange Rates: A Conditional Variance Tale,' *Journal of Business and Economic Statistics*, July 1989, **7:** 295–305.

BAILLIE, R.T., AND T. BOLLERSLEV, 'A Multivariate Generalized ARCH Approach to Modelling Risk Premia in Forward Foreign Exchange Rate Markets,' *Journal of International Money and Finance*, September 1990, **9:** 309–324.

BAILLIE, R.T., AND R.J. MYERS, 'Modelling Commodity Price Distributions and Estimating the Optimal Futures Hedge,' unpublished manuscript, Michigan State University, 1989, forthcoming in *Journal of Applied Econometrics.*

BERA, A.K., AND C. JARQUE, 'Model Specification Tests: A Simultaneous Approach,' *Journal of Econometrics*, October 1982, **20:** 59–82.

BOLLERSLEV, T., 'Generalized Autoregressive Conditional Heteroskedasticity,' *Journal of Econometrics*, April 1986, **31:** 307–327.

BOLLERSLEV, T., 'Modelling the Coherence in Short Run Nominal Exchange Rates: A Multivariate Generalized ARCH Model,' *Review of Economics and Statistics*, 1990, **72:** 498–505.

BOLLERSLEV, T., R.T. CHOU, AND K.F. KRONER, 'ARCH Modeling in Finance: A Review of the Theory and Empirical Results,' unpublished manuscript, Department of Economics, University of Arizona, 1990, forthcoming in *Journal of Econometrics.*

BOLLERSLEV, T., AND R.F. ENGLE, 'Common Persistence in Conditional Variances: Definition and Representation,' unpublished manuscript, J.L. Kellogg Graduate School of Management, Northwestern University, 1989.

BOLLERSLEV, T., R.F. ENGLE, AND J.M. WOOLDRIDGE, 'A Capital Asset Pricing Model with Time Varying Covariances,' *Journal of Political Economy*, February 1988, **96:** 116–131.

BOLLERSLEV, T., AND J.M. WOOLDRIDGE, 'Quasi Maximum Likelihood Estimation and Inference in Dynamic Models with Time Varying Covariances,' unpublished manuscript, MIT, 1990.

BREEDEN, D., 'An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities,' *Journal of Financial Economics*, September 1979, **7:** 265–296.

CECCHETTI, S.G., R.E. CUMBY, AND S. FIGLEWSKI, 'Estimation of the Optimal Futures Hedge,' *Review of Economics and Statistics*, November 1988, **70:** 623–630.

CLAESSENS, S., 'The Optimal Currency Composition of External Debt,' Policy, Planning and Research Working Paper No. 14, World Bank, 1988.

DIEBOLD, F.X., *Empirical Modelling of Exchange Rate Dynamics*, New York: Springer Verlag, 1988.

DIEBOLD, F.X., AND M. NERLOVE, 'The Dynamics of Exchange Rate Volatility: A Multivariate Latent Factor ARCH Model,' *Journal of Applied Econometrics*, January–March 1989, **4**: 1–21.

DORNBUSCH, R., 'Exchange Rates and Prices,' *American Economic Review*, March 1987, **77**: 93–106.

ENGLE, R.F., 'Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of U.K. Inflation,' *Econometrica*, July 1982, **50**: 987–1008.

ENGLE, R.F., AND T. BOLLERSLEV, 'Modelling the Persistence of Conditional Variances,' *Econometric Reviews*, 1986, **5**: 1 – 50, 81–87.

FISCHER, S., 'The Demand For Index Bonds,' *Journal of Political Economy*, June 1975, **83**: 509–534.

GIOVANNINI, A., AND P. JORION, 'The Time Variation of Risk and Return in the Foreign Exchange and Stock Markets,' *Journal of Finance*, June 1989, **44**: 307–325.

HONG, C.-H., The Integrated Autoregressive Conditional Heteroskedastic Model: The Process, Estimation and Monte Carlo Experiment,' unpublished manuscript, UC San Diego, 1988.

HSIEH, D.A., 'Modeling Heteroskedasticity in Daily Foreign Exchange Rates,' *Journal of Business and Economic Statistics*, 1989, **7**: 307–317.

Indikator Ekonomi, Biro Pusat Statistik, Jakarta, Indonesia, various issues.

International Financial Statistics, International Monetary Fund Publication, various issues.

KRAFT, D.F., AND R.F. ENGLE, 'Autoregressive Conditional Heteroskedasticity in Multiple Time Series Models,' UC San Diego Discussion Paper No. 82-23, 1982.

KRONER, K.F., 'Estimating and Testing for Factor ARCH,' unpublished manuscript, UC San Diego, 1987.

KRONER, K.F., AND J. SULTAN, 'Foreign Currency Futures and Time Varying Hedge Ratios,' in S. Ghon Rhee and R.P. Chang, eds, *Pacific-Basin Capital Markets Research*, Volume II, forthcoming, 1991.

LESSARD, D.R., AND J. WILLIAMSON, *Financial Intermediation Beyond the Debt Crisis*, Washington, DC: Institute for International Economics, 1985.

MERTON, R.C., 'Optimal Consumption and Portfolio Rules in a Continuous-Time Model,' *Journal of Economic Theory*, 1971, **3**: 373–413.

PHILLIPS, P.C.B., AND P. PERRON, 'Testing for a Unit Root in Time Series Regressions,' *Biometrika*, 1988, **75**: 335–346.

STULZ, R., 'A Model of International Pricing,' *Journal of Financial Economics*, December 1981, **9**: 383–406.

SVENSSON, L.E.O. 'Optimal Foreign Debt Composition,' unpublished manuscript, World Bank, December 1987.